From epidemic model to quantum mechanics and q-AI

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Overview

Features of quantum mechanics

- Superposition of states
- Entanglement and non-local correlations
- Quantum Coherence
- Quantum Measurement Destroying Quantum State
- Non-cloning and no-deleting theorem
- Path integral approach

Features of classical statistical physics

- Probability of occurrence of many states and processes
- Path integral approach
- Stochastic determinism
- Intrinsic noise

Schroedinger equation

Equations of motion for isolated quantum system are given as

$$\hat{H}\left|\psi(t)\right\rangle = i\hbar\frac{d}{dt}\left|\psi(t)\right\rangle = E(t)\left|\psi(t)\right\rangle, \langle x|\psi(t)\rangle = \psi(x,t) \quad (1)$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x,t) + V(x,t)\psi(x,t) = E(t)\psi(x,t) = i\hbar\frac{d}{dt}\psi(x,t), \quad (2)$$

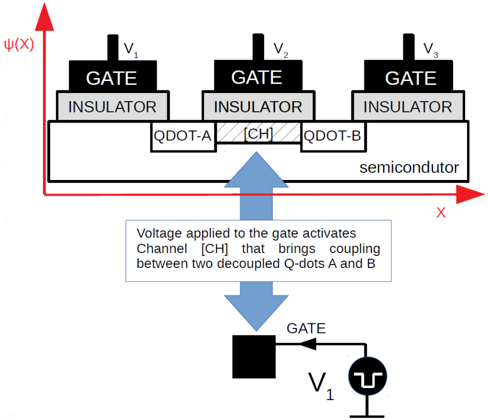
$$\left|\psi(t)\right\rangle = [e^{\frac{1}{i\hbar}\int_{t_0}^{t}\hat{H}(t')dt'}]\left|\psi(t_0)\right\rangle, \int_{-\infty}^{+\infty}\psi(x,t)^*\psi(x,t)dx = \langle\psi|\psi\rangle = 1, \quad (3)$$

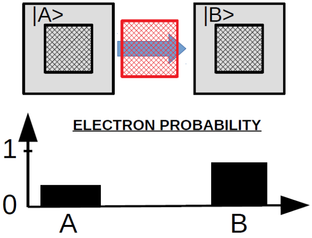
where $|\psi(x,t)|^2$-probability density at point x, $\hat{H}$-Hamiltonian operator, $\psi(x,t) = \psi(x,t)_{Re} + i\psi(x,t)_{Im}$ with $\psi(x,t)_{Re}, \psi(x,t)_{Im} \in R$.

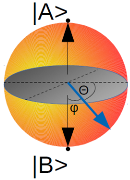
$$|\psi(x,t)|^2 = |\psi_{Re}(x,t)|^2 + |\psi_{Im}(x,t)|^2 = p_{Re}(t) + p_{Im}(t) = p(t) - observed. \quad (4)$$

Here E(t) are system eigenergies that can take both continuous and discrete values. Discrete energy values are unique property of quantum systems. In case of isolated quantum system we have $\hat{H}^\dagger = \hat{H}$ that is not the case of open quantum system (with dissipation) that is interacting with environment (outside world).

Position dependent qubit in chain of coupled quantum dots







Quantum state of qubit |Ψ> is the superposition of |A> and |B> states given relation |Ψ> = a|A> + b|B> and parametrized by 2 Bloch sphere angles Θ and φ

K.Pomorski et al. Cryogenics 2020, K.Pomorski et al. Spie 2020, P.Giounanlis et al. IEEE Access 2019, K.Pomorski Springer 2020.

Tight-binding model and Wannier qubit

For coupled quantum dots we have probability of presence of electron on the left and right quantum dot. In such case one have the localized energy of electron at left q-dot as E_{p1}, on right q-dot E_{p2} and travelling (hopping) energy from left to right q-dot given as $|t_{s12}|$.

$$\begin{pmatrix} E_{p1} & t_{s12} \\ t_{s12}^* & E_{p2} \end{pmatrix} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} = i\hbar \frac{d}{dt} \begin{pmatrix} \psi_1(t) \\ \psi_1(t) \end{pmatrix} = E \begin{pmatrix} \psi_1(t) \\ \psi_1(t) \end{pmatrix} \quad (5)$$

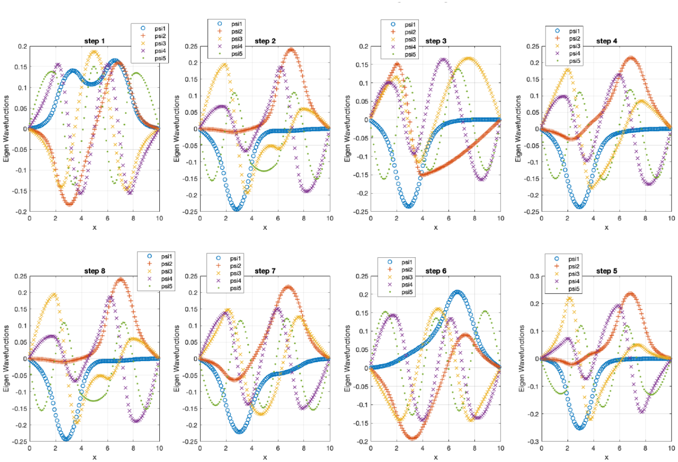
with $E_{p1}(t)$, $E_{p2}(t)$, $t_{s12}(t)$ and $t_{s21}(t)$ given by formulas

$$\psi(x) = \gamma_1 w_L(x) + \gamma_R w_R(x), w_L - maximum\ localized\ wavefunction\ on\ left,$$

$$E_{p1}(t) = \int_{-\infty}^{+\infty} dx w_L^*(x,t)(-\frac{\hbar^2}{2m_e}\frac{d^2}{dx^2} + V(x,t))w_L(x,t),$$

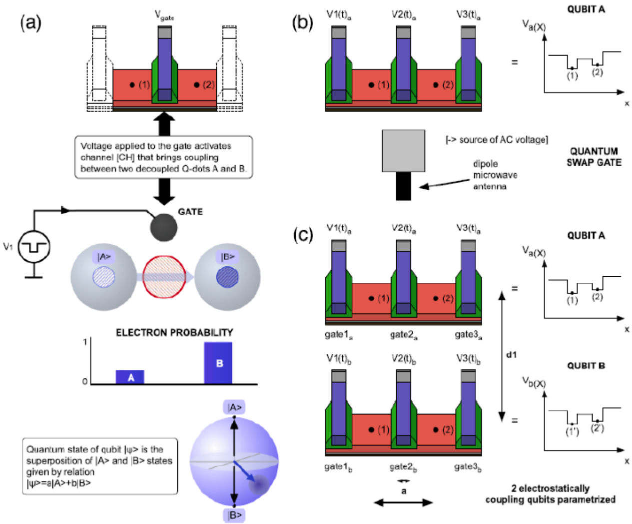
$$E_{p2}(t) = \int_{-\infty}^{+\infty} dx w_R^*(x,t)(-\frac{\hbar^2}{2m_e}\frac{d^2}{dx^2} + V(x,t))w_R(x,t),$$

$$t_{s12(s21)}(t) = \int_{-\infty}^{+\infty} dx w_{R(L)}^*(x,t)(-\frac{\hbar^2}{2m_e}\frac{d^2}{dx^2} + V(x,t))w_{L(R)}(x,t),$$



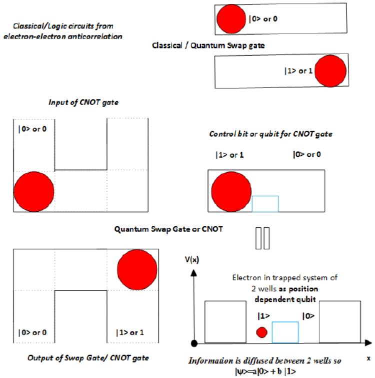
K.Pomorski, Part of the Advances in Intelligent Systems and Computing book series, AISC, Vol. 1289, Springer, 2020

Case of electrostatically interacting Wannier qubits



K.Pomorski et al. Cryogenics 2020

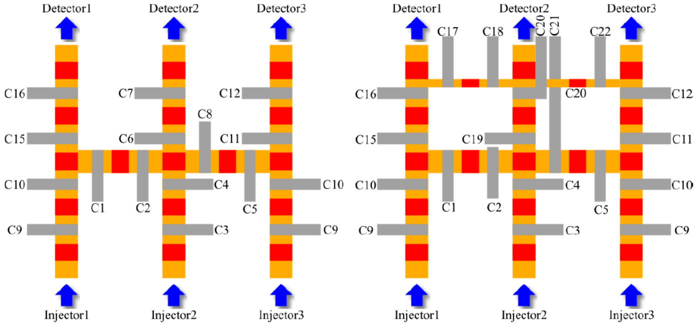
Anticorrelation principle in Q-Logic in Wannier qubits



From two types of electrostatic position-dependent semiconductor qubits to quantum universal gates and hybrid semiconductor-superconducting quantum computer, Spie 2020, Pomorski et al.

K.Pomorski et al. Spie, 2019

Quantum neural network in chain of coupled quantum dots



Concept of quantum and classical single electron neural network in CMOS. It can mimic all quantum universal gates. It can be controlled with voltages applied to C1,..,C12, .., C22 gates. Additionally one can use external magnetic field to control its performance.

Q-Neural Network can mimic any physical system of N bodies.

K.Pomorski, November 2018 as by `https://www.youtube.com/watch?v=mc7gctooccE` (55 min of recording).

Case of 2 coupled Wannier qubits

$$\begin{pmatrix} E_{p1A}+E_{p1B}+\frac{q^2}{d_{1A-1B}} & t_{s(2_B\to 1_B)} & t_{s(2_A\to 1_A)} & 0 \\ t_{s(1_B\to 2_B)} & E_{p1A}+E_{p2B}+\frac{q^2}{d_{1A-2B}} & 0 & t_{s(2_A\to 1_A)} \\ t_{s(1_A\to 2_A)} & 0 & E_{p2A}+E_{p1B}+\frac{q^2}{d_{2A-1B}} & t_{s(2_B\to 1_B)} \\ 0 & t_{s(1_A\to 2_A)} & t_{s(1_B\to 2_B)} & E_{p2A}+E_{p2B}+\frac{q^2}{d_{2A-2B}} \end{pmatrix} \times$$

$$\times \begin{pmatrix} \sqrt{p_I(t)}e^{i\Theta_I(t)} \\ \sqrt{p_{II}(t)}e^{i\Theta_{II}(t)} \\ \sqrt{p_{III}(t)}e^{i\Theta_{III}(t)} \\ \sqrt{p_{IV}(t)}e^{i\Theta_{IV}(t)} \end{pmatrix} =$$

$$= i\hbar\frac{d}{dt}\begin{pmatrix} \sqrt{p_I(t)}e^{i\Theta_I(t)} \\ \sqrt{p_{II}(t)}e^{i\Theta_{II}(t)} \\ \sqrt{p_{III}(t)}e^{i\Theta_{III}(t)} \\ \sqrt{p_{IV}(t)}e^{i\Theta_{IV}(t)} \end{pmatrix}, \textit{K.Pomorski et al., Semiconductor Science and Technology, 2019} \quad (7)$$

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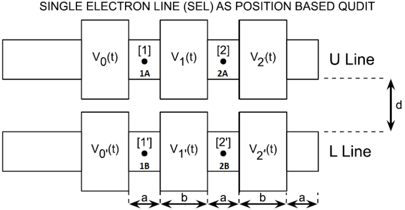

Epidemic model-Equations of motion

$$E_1\left|E_1\right\rangle + E_2\left|E_2\right\rangle = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 2\sqrt{p_1}\frac{d}{dt}(\sqrt{p_1}) \\ 2\sqrt{p_2}\frac{d}{dt}(\sqrt{p_2}) \end{pmatrix} \quad (8)$$

Superposition principle in Classical Epidemic and Tight-Binding Model.

$$E_1\left|E_1\right\rangle + E_2\left|E_2\right\rangle = \begin{pmatrix} E_{p1} & t_{s12} \\ t_{s21} & E_{p2} \end{pmatrix}\begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix} =$$

$$= i\hbar\frac{d}{dt}\begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix} = \begin{pmatrix} e^{i\Theta_I(t)}[i\hbar\frac{d}{dt}\sqrt{p_1} - \hbar\sqrt{p_1}\frac{d}{dt}\sqrt{\Theta_I}] \\ e^{i\Theta_{II}(t)}[i\hbar\frac{d}{dt}\sqrt{p_2} - \hbar\sqrt{p_2}\frac{d}{dt}\sqrt{\Theta_{II}}] \end{pmatrix},$$

$$\begin{pmatrix} E_{p1} + \hbar\frac{d}{dt}\sqrt{\Theta_I} & t_{s12} \\ t_{s21} & E_{p2} + \hbar\frac{d}{dt}\sqrt{\Theta_{II}} \end{pmatrix}\begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix} =$$

$$= i\hbar\frac{d}{dt}\begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix} = \begin{pmatrix} e^{i\Theta_I(t)}[i\hbar\frac{d}{dt}\sqrt{p_1}] \\ e^{i\Theta_{II}(t)}[i\hbar\frac{d}{dt}\sqrt{p_2}] \end{pmatrix}, \quad (9)$$

Equivalently we have

$$\frac{1}{i\hbar}\begin{pmatrix} e^{-i\Theta_I(t)} & 0 \\ 0 & e^{-i\Theta_{II}(t)} \end{pmatrix}\begin{pmatrix} E_{p1}+\hbar\frac{d}{dt}\sqrt{\Theta_I} & t_{s12} \\ t_{s21} & E_{p2}+\hbar\frac{d}{dt}\sqrt{\Theta_{II}} \end{pmatrix}\begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix} = \begin{pmatrix} \frac{d}{dt}\sqrt{p_1} \\ \frac{d}{dt}\sqrt{p_2} \end{pmatrix},$$

and we have

$$2\begin{pmatrix} \sqrt{p_1} & 0 \\ 0 & \sqrt{p_2} \end{pmatrix}\times$$

$$\times\frac{1}{i\hbar}\begin{pmatrix} e^{-i\Theta_I(t)} & 0 \\ 0 & e^{-i\Theta_{II}(t)} \end{pmatrix}\begin{pmatrix} E_{p1}+\hbar\frac{d}{dt}\sqrt{\Theta_I} & t_{s12} \\ t_{s21} & E_{p2}+\hbar\frac{d}{dt}\sqrt{\Theta_{II}} \end{pmatrix}\begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix}$$

$$= 2\begin{pmatrix} \sqrt{p_1} & 0 \\ 0 & \sqrt{p_2} \end{pmatrix}\begin{pmatrix} \frac{d}{dt}\sqrt{p_1} \\ \frac{d}{dt}\sqrt{p_2} \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

as given in ArXiv:2012.09923 by Pomorski (2020).

$$
\begin{aligned}
& 2\begin{pmatrix} \sqrt{p_1} & 0 \\ 0 & \sqrt{p_2} \end{pmatrix} \times \\
& \times \frac{1}{i\hbar}\begin{pmatrix} e^{-i\Theta_I(t)} & 0 \\ 0 & e^{-i\Theta_{II}(t)} \end{pmatrix} \begin{pmatrix} E_{p1} + \hbar\frac{d}{dt}\sqrt{\Theta_I} & t_{s12} \\ t_{s21} & E_{p2} + \hbar\frac{d}{dt}\sqrt{\Theta_{II}} \end{pmatrix} \times \\
& \begin{pmatrix} [\sqrt{p_1}e^{-i\Theta_I(t)}]^{-1} & 0 \\ 0 & \sqrt{p_2}e^{-i\Theta_{II}(t)}]^{-1} \end{pmatrix} \begin{pmatrix} [\sqrt{p_1}e^{-i\Theta_I(t)}] & 0 \\ 0 & \sqrt{p_2}e^{-i\Theta_{II}(t)}] \end{pmatrix} \times \\
& \begin{pmatrix} \sqrt{p_1}e^{i\Theta_I(t)} \\ \sqrt{p_2}e^{i\Theta_{II}(t)} \end{pmatrix} \\
& = 2\begin{pmatrix} \sqrt{p_1} & 0 \\ 0 & \sqrt{p_2} \end{pmatrix} \begin{pmatrix} \frac{d}{dt}\sqrt{p_1} \\ \frac{d}{dt}\sqrt{p_2} \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \quad (11)
\end{aligned}
$$

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