From quantum hardware to quantum AI

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Classical computation vs quantum computation

CLASSICAL LOGIC:

- Sharp Logic [Boolean 0 or 1 State]
- Fuzzy Logic [Based on Continous State between 0 and 1]

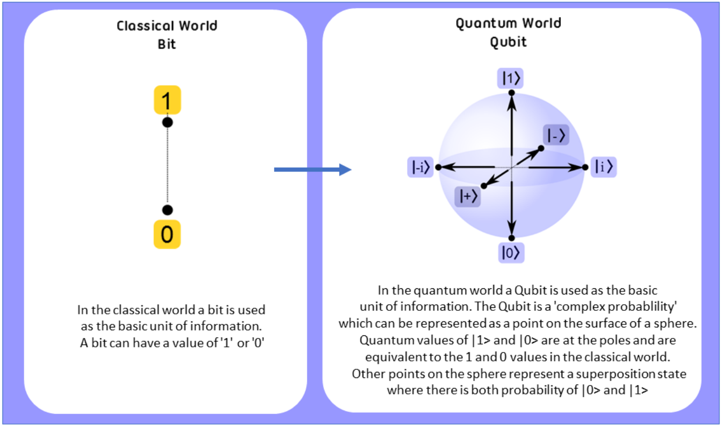
QUANTUM LOGIC:

- Logic BASED on qubits ($|\psi >= \alpha|0> + \beta|1>$), where α and β are complex valued with condition $1 = |\alpha|^2 + |\beta|^2$ what gives $\alpha = cos(\Theta)e^{i\gamma}, \beta = sin(\Theta)e^{i\delta}$

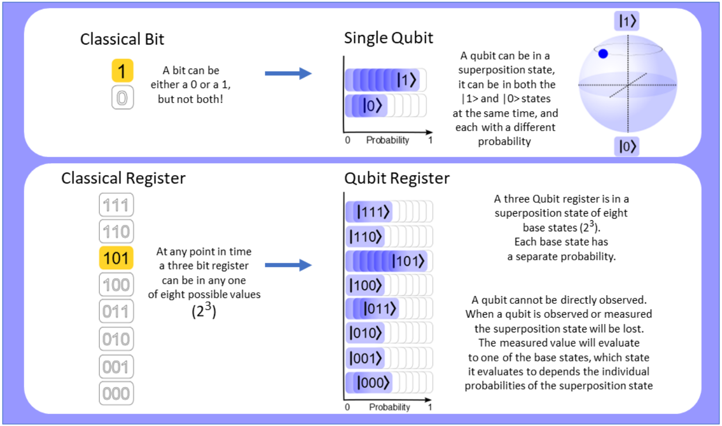
It is important to note that qubit state can be always written as

$$|\psi >= \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = e^{i\gamma}[cos(\Theta) \begin{pmatrix} 0 \\ 1 \end{pmatrix} + sin(\Theta)e^{i(\delta-\gamma)} \begin{pmatrix} 1 \\ 0 \end{pmatrix}].$$

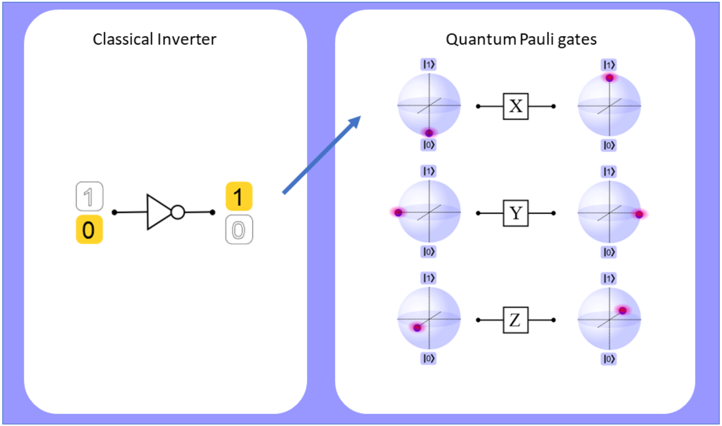
Classic vs Quantum Computing Basics



Qubit Register Superposition

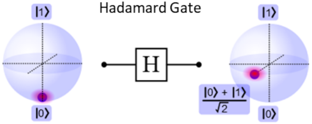


Qubit Inverter Gates



Quantum Hadamard and CNOT gates

Quantum
Hadamard Gate

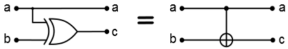


The Hadamard gate transforms a |1> or |0> into an superposition state with equal probability of |0> or |1>. There is no equivalent function in the classical world

Classical Exor Gate

a b	c
0 0	0
0 1	1
1 0	1
1 1	0

Quantum CNOT Gate



ab	ac
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

Quantum gates must be reversible and have an equal Number of inputs and outputs

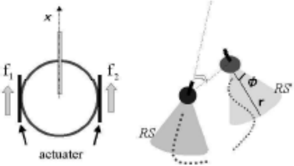
Computation as physical process

In all cases the computation is the results of physical evolution of the given system that is implemented in some technology. At first stage we set certain initial conditions of the system and we allow them to evolve what is equivalent of performing the algorithmic steps. After certain time system is achieving its final state. Then we perform the readout or measurement on certain sections of physical system that we name registors.In such way we determine the computational result. System can evolve in dissipative and sometimes in non-dissipative way so there is presence of friction and entropy is usually increasing. By delivering energy to the system the information entropy might also decrease. For example filtering the image might bring lower entropy of the picture once it is being processed. It is valid for classical and quantum computers.

Concept of programmable matter

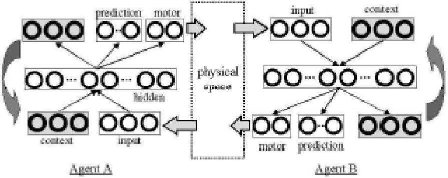
Programmable matter is matter which has the ability to change its physical properties (shape, density, moduli, conductivity, optical properties, etc.) in a programmable fashion, based upon user input or autonomous sensing. Programmable matter is thus linked to the concept of a material which inherently has the ability to perform information processing.

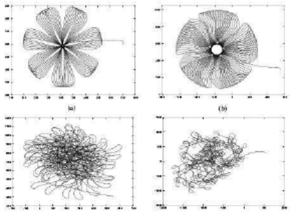
Braitenberg vehicles



Adaptability and Diversity in Simulated Turn-taking Behaviour Hiroyuki Iizuka Takashi Ikegami, $\rightarrow$ Quantum Braitenberg vehicles with use of time-depedent Schroedinger equation+finite state machine??? as next stage towards Q-Alife???? Or shall we consider the quantum ants ???

Ikegami Braitenberg vehicles

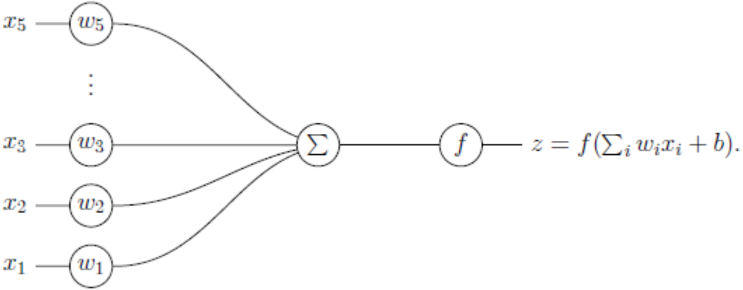




General defintion of AI

From that point of view AI or embodied AI is also physical evolution with use of concept of programmable matter. AI is programmable matter that is able to interact with enviroment in dynamical way. Embodied AI is special version of AI where the interaction of enviroment and artificial evolution brings the emergence of new properties.

For a single neuron z depending on x = (x1, . . . , xn), the mathematical operation can thus be visualized as it is depicted.



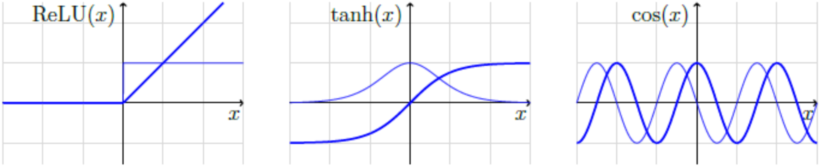
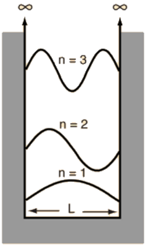


Figure: Non-linear activation functions used in Artificial Neural Networks.

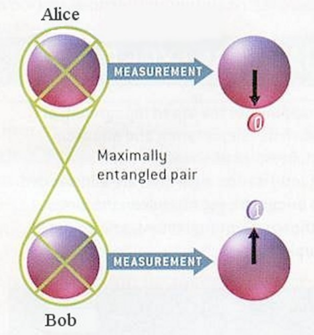
Schroedinger equation

$$\frac{-\hbar^2}{2m}\nabla^2\Psi(\mathrm{r}) + V(r)\Psi(\mathrm{r}) = E\Psi(\mathrm{r})$$

Kinetic Energy + *Potential Energy* = *Total Energy*

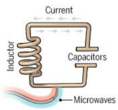


Quantum entanglement



Quantum state collapses after measurement.
$|\psi>=\frac{1}{2}(|\uparrow\downarrow> \pm |\downarrow\uparrow>), \rightarrow |\psi_1>=(|\uparrow\downarrow>)$ [collapse of wavefunction after measurement]!!!!

Superconducting loops



A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.

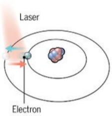
Longevity (seconds)	**0.00005**
Logic success rate	**99.4%**
Number entangled	**9**

Company support
Google, IBM, Quantum Circuits

Pros
Fast working. Build on existing semicon-ductor industry.

Cons
Collapse easily and must be kept cold.

Trapped ions



Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in super-position states.

Longevity (seconds)	**>1000**
Logic success rate	**99.9%**
Number entangled	**14**

Company support
ionQ

Pros
Very stable. Highest achieved gate fidelities.

Cons
Slow operation. Many lasers are needed.

Silicon quantum dots

These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.

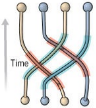
Longevity (seconds)	**0.03**
Logic success rate	**~99%**
Number entangled	**2**

Company support
Intel

Pros
Stable. Build on existing semiconductor industry.

Cons
Only a few entangled. Must be kept cold.

Topological qubits



Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures. Their braided paths can encode quantum information.

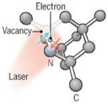
Longevity (seconds)	**N/A**
Logic success rate	**N/A**
Number entangled	**N/A**

Company support
Microsoft, Bell Labs

Pros
Greatly reduce errors.

Cons
Existence not yet confirmed.

Diamond vacancies



A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

Longevity (seconds)	**10**
Logic success rate	**99.2%**
Number entangled	**6**

Company support
Quantum Diamond Technologies

Pros
Can operate at room temperatu

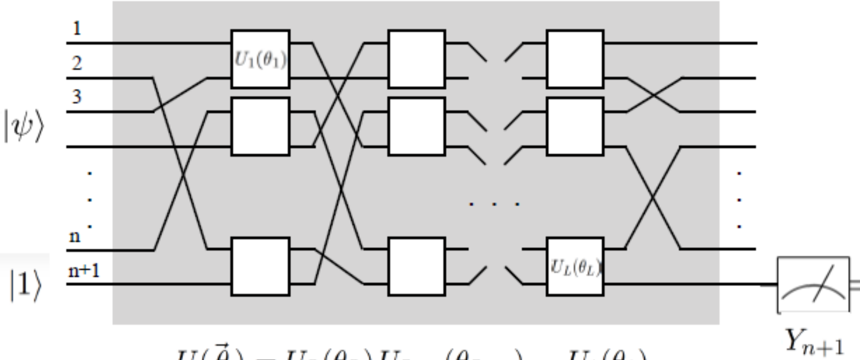
Cons
Difficult to entangle.

Key features of Quantum Mechanics and Quantum Technologies

1. Massive parallelism occurs in isolated quantum system
2. Non-sharp trajecories and lack of full determinism.
3. Quantum metrology and quantum sensing is the best perception.
[However the object under observation is changing what also affects the measurement apparatus]
4. No-cloning and no-deleting theorem.
5. Quantization of physical quantities as energy, momentum, etc ...
6. Non-locality as occurence of entanglement that is spooky action on the distance.
7. Occurence of teleportation.
8. Some analogies of QM with Classical Statistical Mechanics .

During a lecture in the early 1980s, Richard Feynman proposed the concept of simulating physics with a quantum computer (Feynman 1982). He postulated that by manipulating the properties of quantum mechanics and quantum particles one could develop an entirely new kind of computer, one that could not be described by the classical theory of computation with Turing machines. Nature does not explicitly perform the calculations to determine the speed of a ball dropped from a tall building; it does so implicitly. Extending this line of thinking, Feynman wondered if one could harness the complex calculations nature performs intrinsically in quantum mechanics to design a computer with more computational power.

General scheme of quantum neural network



$$U(\vec{\theta}) = U_L(\theta_L)\, U_{L-1}(\theta_{L-1}) \ldots U_1(\theta_1)$$

Comparison of Wavefunction vs ANN

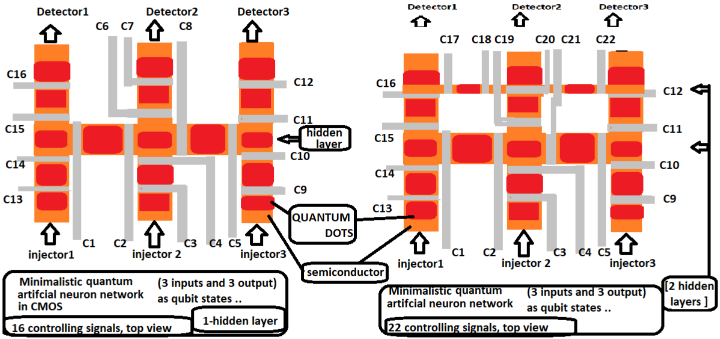
Quantum mechanics	Neural Networks
wave function	neuron
Superposition (coherence)	interconnections (weights)
Measurement (decoherence)	evolution to attractor
Entanglement	learning rule
unitary transformations	gain function (transformation)

As from [4].

Classical neural networks		Quantum associative memory	
Neuronal State	$x_i \in \{0,1\}$	Qubit	$\lvert x\rangle = a\lvert 0\rangle + b\lvert 1\rangle$
Connections	$\{w_{ij}\}_{ij=1}^{p-1}$	Entanglement	$\lvert x_0 x_1 \ldots x_{p-1}\rangle$
Learning rule	$\sum_{s=1}^{p} x_i^s x_j^s$	Superposition of entangled states	$\sum_{s=1}^{p} a_s \lvert x_0^s \ldots x_{p-1}^s\rangle$
Winner search	$n = \max_i \arg(f_i)$	Unitary transformation	$U : \psi \rightarrow \psi'$
Output result	n	Decoherence	$\sum_{s=1} a_s \lvert x^s\rangle \Rightarrow \lvert x^k\rangle$

As from [4].

Quantum neural network in coupled q-dots



Concept of fidelity and quantum fidelity

Given two random variables X, Y with values (1...n) and probabilities p = (p1...pn) and q = (q1...qn). The fidelity of X and Y is defined to be the quantity

$$F(X, Y) = \sum_i \sqrt{p_i q_i}. \tag{2}$$

Given two density matrices ρ and σ, the fidelity is defined by[2]

$$F(\rho, \sigma) = \left(\mathrm{Tr} \sqrt{\sqrt{\rho}\sigma\sqrt{\rho}} \right)^2. \tag{3}$$

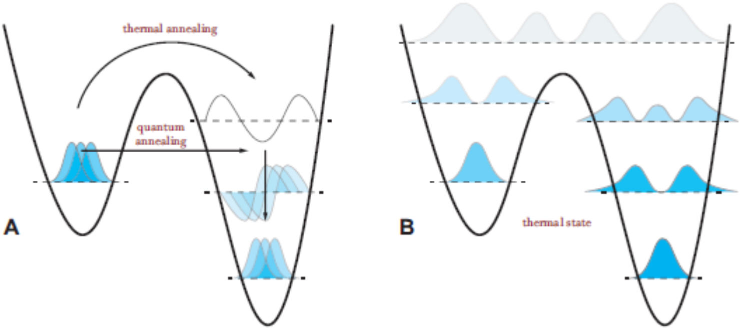
There are many other measures of the distance!!!!

Bures distance

Fidelity can be used to define metric on the set of quantum states, so called Bures distance:

$$DB(X, Y) = 2 - 2F(\rho, \sigma). \tag{4}$$

Classical vs quantum annealing



In most general case the eigenstate of quantum system can be written in the following way at given time instant t.

$$|\psi(t)>= a_0(t)|0(t)> +a_1(t)|1(t)>< 1(t)| + ..a_{n1}(t)|n1(t)> + \\ +(\int_{q1(t)}^{q2(t)} g(e1,t)de1)|e1(t)> . \quad (5)$$

The coefficients $a_0, .., a_n$ fulfill the normalization condition $1 = |a_0|^2 + ..|a_n|^2 + \int_{q1(t)}^{q2(t)} |g^2|de_1$. Dynamics of quantum state gives

$$i\hbar\frac{d}{dt}|\psi(t)>= H(t)|\psi(t)>= \hat{E}(t)|\psi(t)> . \quad (6)$$

and can be written in discrete form as

$$|\psi(t+\Delta t)>= |\psi(t)> +\frac{-i\Delta t}{\hbar}(H(t+\Delta t)|\psi(t)>). \quad (7)$$

$$H(t) = E_0(t)|0(t) >< 0(t)| + E_1(t)|1(t) >< 1(t)| + .. +$$
$$+E_n(t)|n(t) >< n(t)| + \int_{q1(t)}^{q2(t)} f(e1,t)de1|e1(t) >< e1(t)|. \quad (8)$$

$$a_0(t+\Delta t) = (< 0(t+dt)|0(t) > a_0(t) + < 0(t+dt)|1(t) > a_1(t) + ..$$
$$+ < 0(t+dt)|n(t) > a_n(t)) + ..$$
$$a_1(t+\Delta t) = (< 1(t+dt)|0(t) > a_0(t) + < 1(t+dt)|1(t) > a_1(t) + ..$$
$$+ < 1(t+dt)|n(t) > a_n(t)) + ..$$
$$..$$
$$a_n(t+\Delta t) = (< n(t+dt)|0(t) > a_0(t) + < n(t+dt)|1(t) > a_1(t) + ..$$
$$+ < n(t+dt)|n(t) > a_n(t)) + ..$$

In 2-body system
$H = -\frac{\hbar^2}{2m}([\frac{d^2}{dx_1^2} \times I] + [I \times \frac{d^2}{dx_2^2}]) + \frac{e^2}{4\pi\epsilon_0|x_1 - x_2|} + V_1(x_1, t) + V_2(x_2, t)$
with $\psi(x_1, x_2) = \psi(x_0 + i\Delta x, x_0 + j\Delta x) = \psi_{i,j}$ and Euler scheme for 2nd derivative. Initial state is
$\psi(x, y, t_0) = \sum_{i,j} a_{i,j}\psi_i(x_1, t_0)\psi_j(x_2, t_0)), \sum_{i,j} |a_{i,j}|^2 = 1$ is linear combination of non-interacting free particles with E_i, E_j energies.

$$(-\frac{\hbar^2}{2m}\frac{d^2}{dx_A^2}+\lambda\frac{\int_{-\infty}^{+\infty}e^2\psi_B(x)\psi_B^\dagger(x)dx}{4\pi\epsilon_0|x_A-x|}+V_A(x_A))\psi_A(x_A)=E_A\psi_A(x_A)$$

$$(-\frac{\hbar^2}{2m}\frac{d^2}{dx_B^2}+\lambda\frac{\int_{-\infty}^{+\infty}e^2\psi_A(x)\psi_A^\dagger(x)dx}{4\pi\epsilon_0|x_B-x|}+V_B(x_B))\psi_B(x_B)=E_B\psi_B(x_B)$$

$$\int_{-\infty}^{+\infty}\psi_A^\dagger(x_A)H_{B-eff}\psi_B(x_A)dx_A+$$
$$+\int_{-\infty}^{+\infty}\psi_B^\dagger(x_B)H_{B-eff}\psi_B(x_B)dx_B=<H_{total}>$$

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